I think you please remember these figures okay which is very interesting figures for a fluid mechanic student point of view that when you have this a flow passing over the plate, you can anticipate it that you will have a viscous effect zone which is called boundary layer formations, there is a zone where there is large gradient of velocity vectors, the velocity will start from 0 to a large gradient will be there.

Since, there is a large gradient of velocity variation is there as I said it earlier that velocity vector change from one point to other point, so the fluid particles will not go straight line, they will start rotating it, so that way this figure is retreating that when the fluid particles entered here, there are large velocity gradients are there, the turbulence is there, so the boundary layer formations are the zone where viscous effect dominates okay.

Those regions you will see the vorticity would be there or you just have the cross product of the del and the V, will show the vorticity and the graphically, you can see the face of this one's okay, it is just a vorticity okay, it is a rotation, the fluid particles will go under the rotations but just the particles which is much above this okay, where there is no vortex, you can see the face of this one's okay, they are just same, there is no rotation.

So, then we tell it irrotational fluid, this is the outside of the boundary layers, you can see that fluid particles are moving it or the virtual fluid balls are moving it without any rotations but within the boundary layer formations, the small regions near to a surface, you will see there is a change of the velocity gradients; the drastic change of the velocity gradient and those the regions; a thin region is called boundary layer.

There; because there is large variations of velocity factors induces the fluid particles to be start rotating it, once you start rotating it so, once particles start rotating, other one will start rotating so, all the particles will start rotating it and that what it is having formations of Eddies formations and all which we will discuss in the later on in a pipe flow chapters.

But you try to understand it like these figures, you please try to visualize these figures and see there are will be the regions where the rotations will be very dominate and there is a regions you will not have a rotations that much of a dominate, it can consider is irrotational zones, where the fluid particles will not go for any rotational, they will have the translations, there may have the linear shear strain formations, may have the linear strain formations but they will not have a rotational activity.

So, that what we measure in terms of vorticity, so please do not have a very confusions between the vorticity and angular vector because vorticity is easy to define is a cross product between the delta and the V where is when you talk about angular rotations , we have half of that so, it is very easy the people who are not looking the angular rotations, they are looking it in terms of how the vorticity is playing it or vortex formations happening it, they talk about at the vorticity level not at the rotations level.

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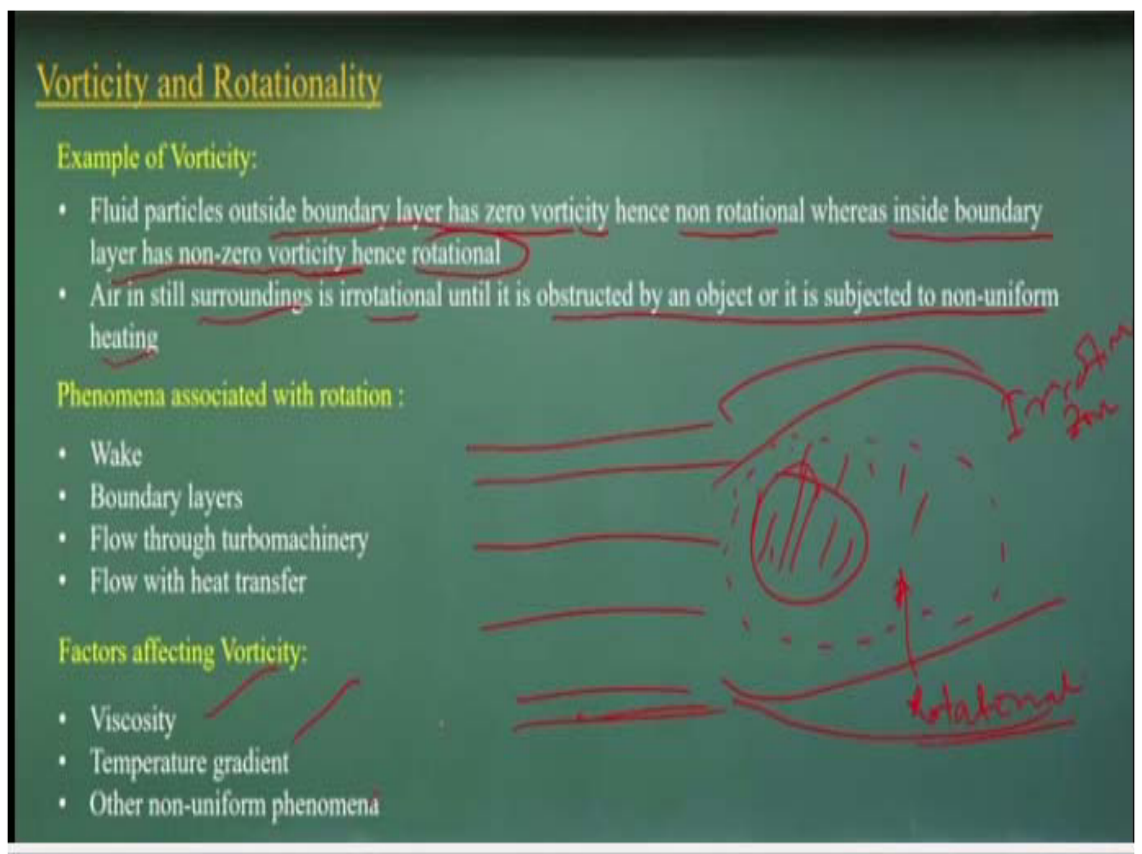

These are very examples as I said it that when the fluid particles outside the boundary layer has a zero vorticity non-rotational zone but inside the boundary layers, it will have non zero vorticity, so flow is a rotational, air is still surrounding the irrotational but it is obstructed by object or these things, so air flow wherever is that if there is no obstructions, so it will be the irrotational.

But when you are start putting a obstructions like a any cylinder curl type of subject, so you will see that very close to these regions, will have a lot of the rotational activities but the outside these are irrotational zone, so you try to understand it how is the formations and this is the formations which is called the boundary layers and (()) (36:26) formations, it happens for flow through the turbo machinery, the wake formations and all and which causes viscosity temperature gradient and the non-uniform phenomena.

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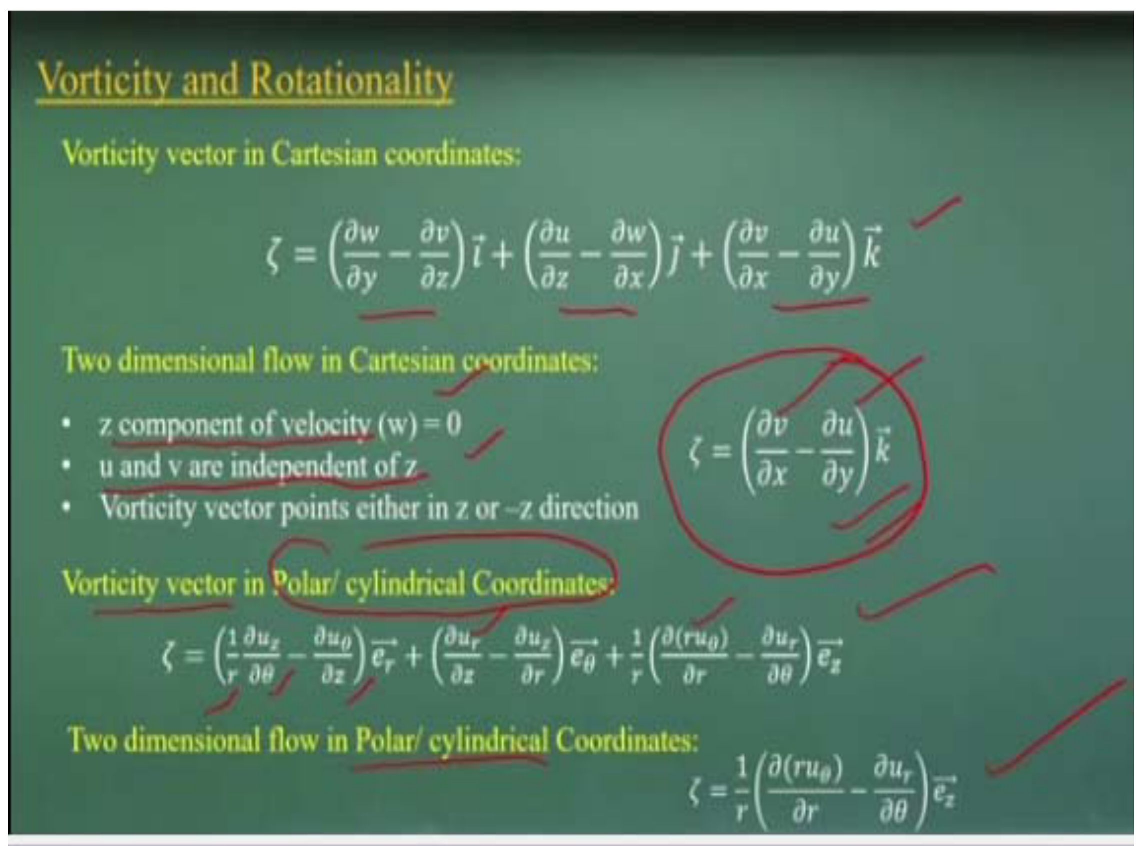

Now, in terms of vorticity vector coordinates if I put it in an i, j, k, I will have this component, if I am looking a 2 dimensional flow, the flow does not have the z component okay and u and v are independent to z, then I have only one components, so many of the flow you know it we can define in terms of 2 dimensional flow, it is easy to visualize, easy to understand it okay, as compared to the 3 dimensional flow.

Vorticity vector in Cartesian coordinates:

$$\zeta = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

Two dimensional flow in Cartesian coordinates:

- z component of velocity (w) = 0
- u and v are independent of z
- Vorticity vector points either in z or –z direction

$$\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

So, that is what we can see the vorticities can define in terms of the scalar velocity gradient component, partial gradient component in x and y directions and the k indicating the directions where the vorticity directions is the z directions. So, vorticity vectors also can retain in terms of polar and cylindrical coordinate systems for 3 dimensional and also the 2 dimensional's.

Vorticity vector in Polar/ cylindrical Coordinates:

$$\zeta = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\overrightarrow{e_r} + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\overrightarrow{e_\theta} + \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\overrightarrow{e_z}$$

Two dimensional flow in Polar/ cylindrical Coordinates:

$$\zeta = \frac{1}{r}\left(\frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\overrightarrow{e_z}$$

So, those are interested in high level of fluid mechanics, please look at the similar way.

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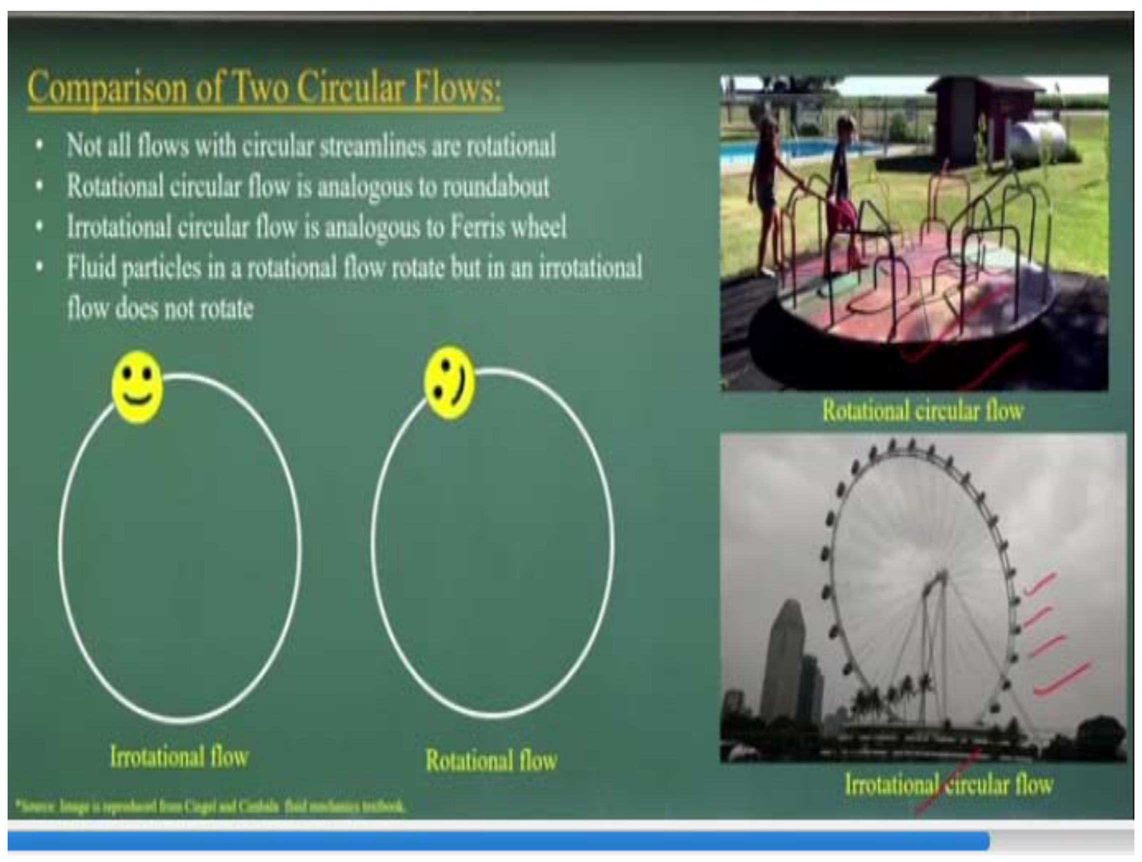

Now if you look at these figures, you can understand it which is irrotational, which is a rotational okay and the figures which is there in here you can look it which is a rotational, which is a irrotational okay, because of that you enjoy this the wheel okay because of its irrotational circular flow, so you just sit it, you enjoy it because it is irrotational but this is the rotational circular flow.

So, that is what is happens it and these figures what is there very interesting figures drawn by my students and you can really visualize these figures and I do believe it if you visualize these figures will never forget what is difference between irrotational and rotational flow. So, with this I can start solving the 2 example problems and concludes today's lectures.

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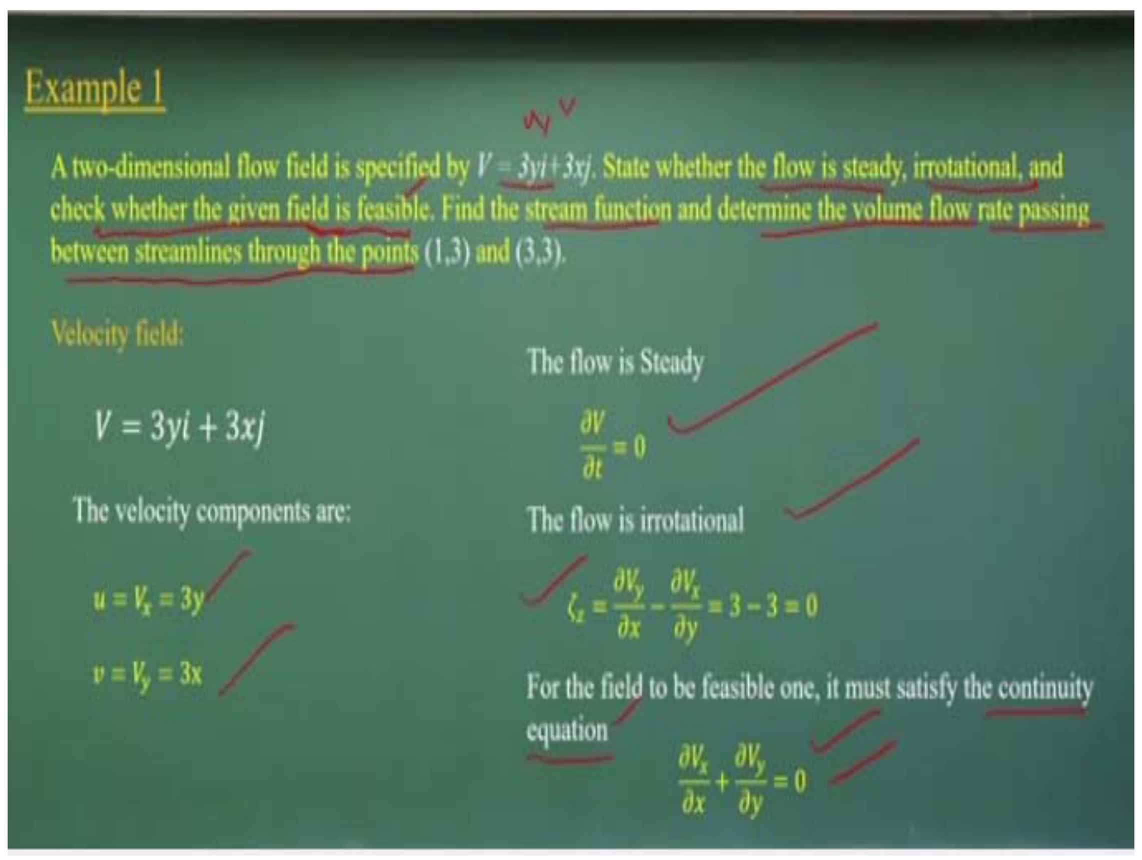

So, if you look at the first point that I am looking a there is a 2 dimensional velocity field state whether the flow is steady, irrotational and check whether the given field is feasible that means, whether the mass conservations happens or not okay, the or the flow is incompressible and mass conservation principles will rotate. Then, we are talking about the stream functions and determining the volume flow rate passing through between the stream line passing through these 2 points okay.

[A two-dimensional flow field is specified by *V = 3yi+3xj*. State whether the flow is steady, irrotational, and check whether the given field is feasible. Find the stream function and determine the volume flow rate passing between streamlines through the points (1,3) and (3,3).]

Even if I have not discussed about the stream function but I will just introduce you what is the stream functions and based on that, you can always think how to get it the stream functions variability. Now, the flow is steady because you can easily locate this u and v component does not have any time component, so the steady flow; flow is irrotational, you can find out the velocity gradients and find out the flow is irrotational.

Velocity field:

$$V = 3yi + 3xj$$

The velocity components are:

$$u = V_x = 3y$$

$$v = V_y = 3x$$

The flow is Steady

$$\frac{\partial V}{\partial t} = 0$$

The flow is irrotational

$$\zeta_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = 3 - 3 = 0$$

For the field to be feasible one, it must satisfy the continuity equation

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

And from the continuity equations or the linear volumetric strain rate functions, you can find substitute these values, you will see that this is equal to 0, that means the flow field is possible one.

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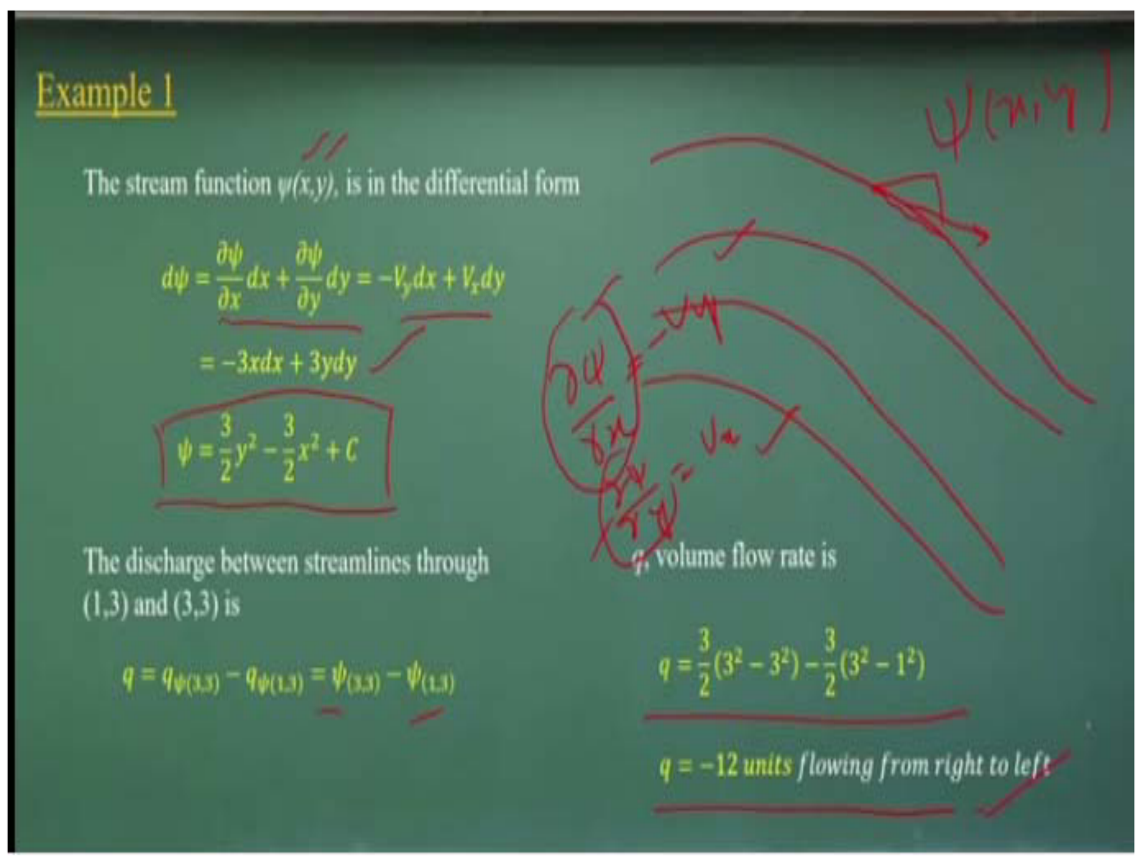

Now, coming back to the stream functions what we define it okay, if you remember it, if these are my stream functions, which varies in x and y, I am talking about the steady stream lines okay, x and y directions as you know it, the stream functions change in the x directions okay, should be equal to – Vy and stream function changing in y directions should be equal to the Vx,

The stream function *ψ(x,y),* is in the differential form

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = -V_y dx + V_x dy$$

$$= -3xdx + 3ydy$$

$$\psi = \frac{3}{2} y^2 - \frac{3}{2} x^2 + C$$

This what you can physically interpret it without having this thing that the tangential component of the velocity factor that what is represent as these functions.

So, considering this the definitions of the stream functions, you can establish this is what the functions behaviour relationship between the stream functions gradient and all, so if as these functions is 2 independent variables, we can define like this substituting this value, we get a stream functions. As you know the stream functions, you can find out the discharge between them which is the difference on that and substitute it here you will get a 12 units.
The discharge between streamlines through (1,3) and (3,3) is

$$q = q_{\psi(3,3)} - q_{\psi(1,3)} = \psi_{(3,3)} - \psi_{(1,3)}$$

q, volume flow rate is

$$q = \frac{3}{2}(3^2 - 3^2) - \frac{3}{2}(3^2 - 1^2)$$

$$q = -12\ units\ flowing\ from\ right\ to\ left$$

Please I can encourage you to read more on the stream functions and all similar type of questions comes in gate or engineering service.

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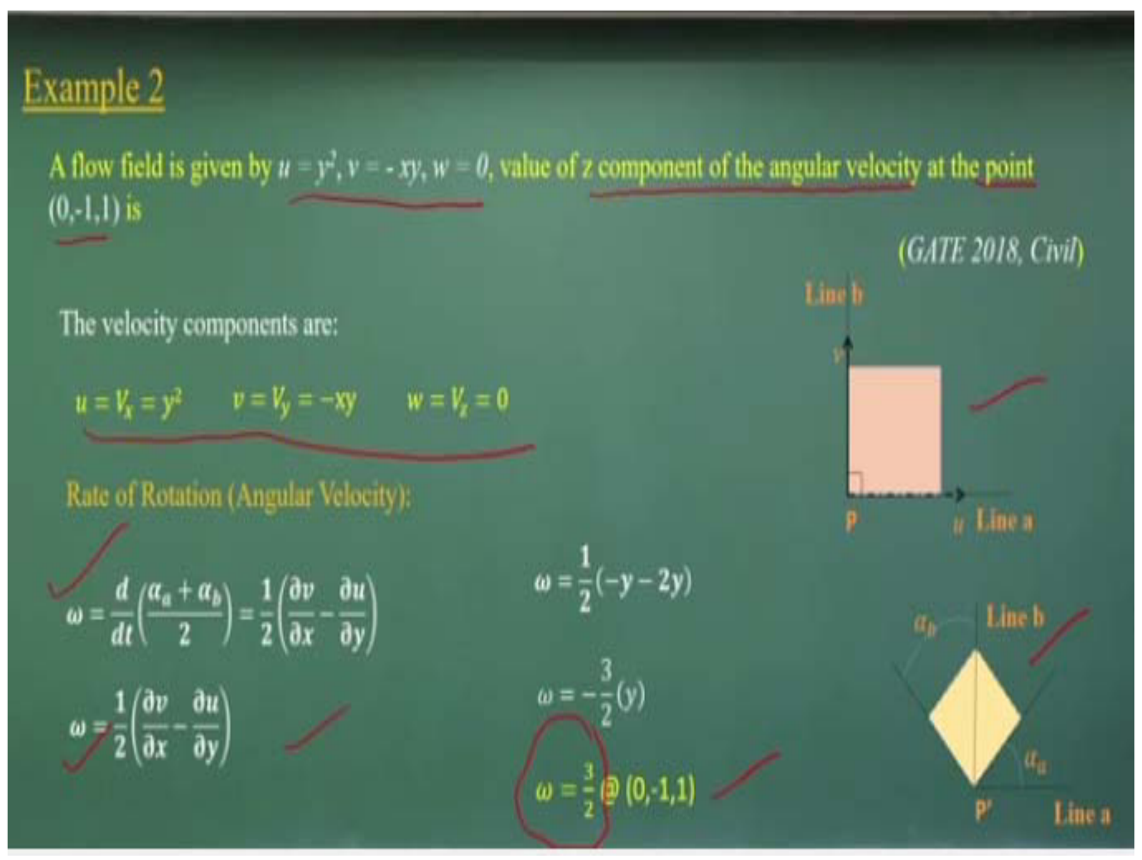

[A flow field is given by $u = y^2$, $v = - xy$, $w = 0$, value of z component of the angular velocity at the point (0,-1,1) is]

Now, coming to second example is a too easy examples as compared to the first one which is a gate 2018 civil engineering questions here, u, v, w components are given it okay, compute the z component of angular velocity at the point, so the point is also given, the x, y, z coordinate is given it, you have to find out the z component of angular velocity that is again the sketch of these figures which is giving alpha and alpha B, you can find u, v, w.

$$u = V_x = y^2$$

$$v = V_y = -\text{xy}$$

$$w = V_z = 0$$

Then, you know this relationship between these things and you just substitute it, you will get it which value is comes out to be 3/2, so the basically you try to understand it, if you find out the scalar components differentiate these scalar; partial differentiations you would do it with respect to y, x or y, then you just substitute it, then you can get what will be the angular rotations in the z directions which is here.

Rate of Rotation (Angular Velocity):

$$\omega = \frac{d}{dt}\left(\frac{\alpha_a + \alpha_b}{2}\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega = \frac{1}{2}(-y - 2y)$$

$$\omega = -\frac{3}{2}(y)$$

$$\omega = \frac{3}{2} \text{ @ } (0, -1, 1)$$

And after that you substitute the coordinate x, y and z at that point, then you will get what will be that.

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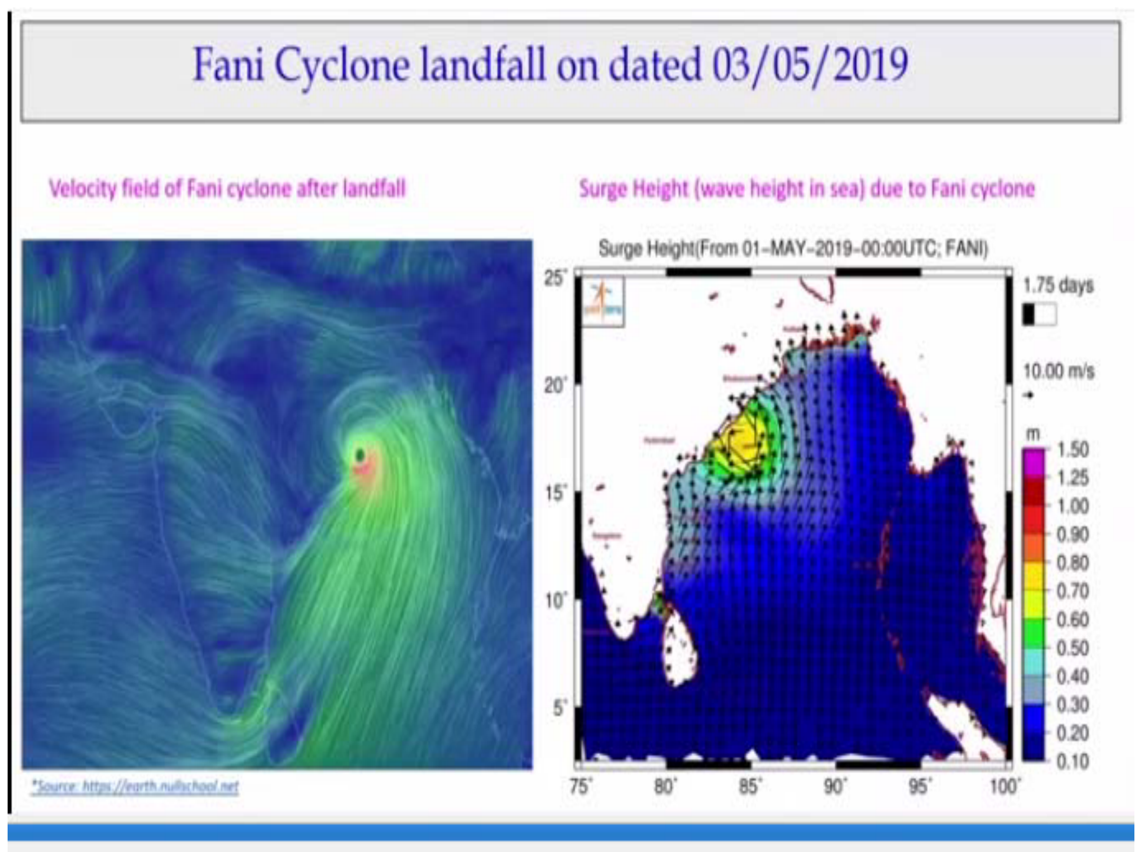

So, with this let us start these figures if you can look it and as its; as I said it is very real time, today is 3rd May 2019 and today is the cyclone is passing through the state of Odisha and you can see these big vortices how it is moving it and how the velocity vectors, the vortex is moving it and how it is causing extreme rainfall events and how it has make it almost the coastal wealth is stands still no flights are running, no trains are running it.

So, these are all it is a possible now because of the high level of fluid mechanics software's we have and which is running at different levels to predict the wind velocities, predict the vorticities, predict all these processing can really see it and also predict the surge height all it is possible now because of our in-depth knowledge of the fluid mechanics, the huge of computational fluid dynamics.

And that is the reason, it is easy to for us to predict a flood or predict the cyclones at the accuracy of is just were one hours difference of accuracy, we can predict at what time it will come it, where it will come it, all these are possible because our knowledge of fluid mechanics and because of our capability to simulate the very complex fluid flow problems in a weak area like if we look at what could be the large area at the global scale also, it is a possible to predict it.

And that is what today's we are able to predict the cyclones even if an accuracy of less than half an hour's, so we can exactly tell it at what time it is going to come it way, so it is all possible and how will be the wind velocity and what will be the wind directions all we can get the path of the cyclone tracks and all these are all possible because of our knowledge of fluid mechanics and our knowledge how to solve this complex fluid flow problems using computer (()) (45:37) or CFD solver.

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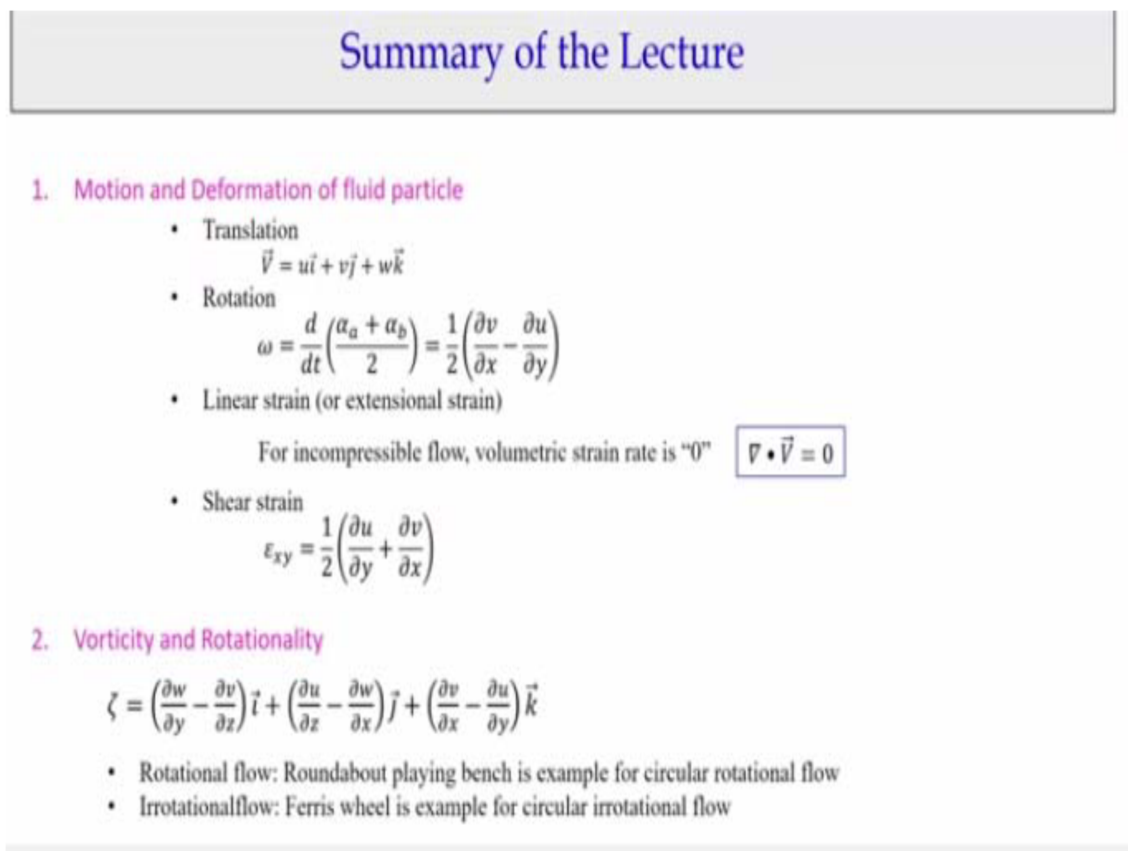

1. Motion and Deformation of fluid particle

   Translation

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

   Rotation

$$\omega = \frac{d}{dt}\left(\frac{\alpha_a + \alpha_b}{2}\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

   Linear strain (or extensional strain)

   For incompressible flow, volumetric strain rate is "0"

$$\nabla \bullet \vec{V} = 0$$

   Shear strain

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

2. Vorticity and Rotationality

   $\zeta = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$

- Rotational flow: Roundabout playing bench is example for circular rotational flow
- Irrotational flow: Ferris wheel is example for circular irrotational flow

With this, let me conclude it, these are all the summary which is listing all the equations, so just have a really it is a good lecture to look at this vorticity, it is a smaller scale to the bigger scale with the cyclonic patterns, with this let us have a; give you a thankful for this lecture, thank you.